BRIEF COMMUNICATIONS

NONSTATIONARY RADIATIVE INTERACTION

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The nonstationary radiative interaction of two infinite plates is considered. A numerical analysis of the thermal stabilization processes is given.

Dynamic transfer processes, which take place very rapidly under high temperature gradients, often manifest themselves in the form of nonstationary thermal interaction. In this type of interaction it is possible to define the initial condition as being the moment at which separate sections of the medium, which have different temperature levels, come into thermal contact.

The form of the thermal contact is determined by the nature of heat transfer at the contact, and also by the structure of the interacting media.

The simplest example of such "incomplete" thermal contact is the nonstationary thermal interaction of two infinite plates in a high vacuum; in this case, the main form of heat transfer is radiation (obeying the Stefan-Boltzmann law), and the resulting heat transfer is

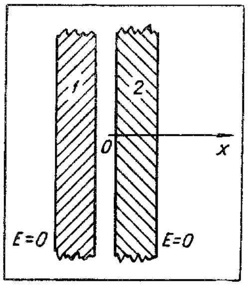

Fig. 1. Radiative interaction of plates.

defined by the Christiansen formula [1]. In convective heat transfer a thermal interaction occurs between the oncoming medium and the surface of the body (Newton's law). The presence of so-called "intermittence" in the contact region evidently tends to reduce the interaction effect.

In the general case, the coefficient for the heat transfer occurring in this type of interaction must be a nonstationary characteristic of the heat-transfer process.

However, we can expect that similar conditions are realized either in the initial period of interaction of the bodies or for the thermal interaction of media with small volumes, which have specific heats of the same order.

For this reason, a detailed analysis is given below for the radiative interaction of two plane-parallel plates of finite thickness, which have different thermophysical properties.

Figure 1 shows a diagram of the thermal interaction between plates. The space between bodies 1 and 2 is represented arbitrarily to show that nonstationary

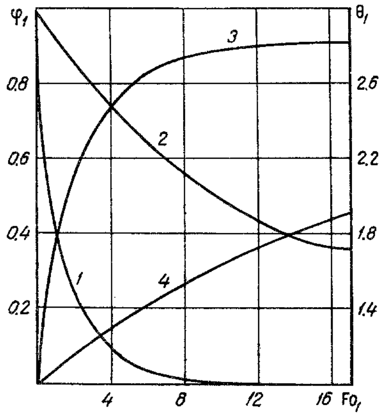

Fig. 2. Thermal stabilization by radiation: 1 and 3 apply respectively to φ_1 and θ_1 for $\sigma_{12} = \sigma_0$; 2 and 4 apply respectively to φ_1 and θ_1 for $\sigma_{12} = 3.97 \cdot 10^{-9}\ \mathrm{W/m^2 \cdot deg^4}$.

interaction takes place according to the laws governing radiative heat transfer between bodies separated by a diathermal medium.

The statement of the problem which was considered in [1] is written as

$$\frac{\partial^2 T_1(x,\ \tau)}{\partial x^2} = \frac{1}{a_1}\,\frac{\partial T_1(x,\ \tau)}{\partial \tau}, \quad -R \leqslant x \leqslant 0,$$

$$\frac{\partial^2 T_2(x,\ \tau)}{\partial x^2} = \frac{1}{a_2}\,\frac{\partial T_2(x,\ \tau)}{\partial \tau}, \quad 0 \leqslant x \leqslant R; \tag{1}$$

$$\lambda_1 \left.\frac{\partial T_1}{\partial x}\right|_{x=-R} = 0,$$

$$\lambda_1 \left.\frac{\partial T_1}{\partial x}\right|_{x=0} = \sigma_{12}\left[T_2^4(0,\ \tau) - T_1^4(0,\ \tau)\right] = E_1(0,\ \tau),$$

$$\lambda_2 \left.\frac{\partial T_2}{\partial x}\right|_{x=R} = 0,$$

$$-\lambda_2 \left.\frac{\partial T_2}{\partial x}\right|_{x=0} = \sigma_{21}\left[T_1^4(0,\ \tau) - T_2^4(0,\ \tau)\right] = E_2(0,\ \tau); \tag{2}$$

$$T_1(x,\ 0) = T_1, \quad T_2(x,\ 0) = T_2, \tag{3}$$

and is solved rigorously by numerical analysis.

The formal solution to the problem, obtained in [1] and described in terms of the temperature in an

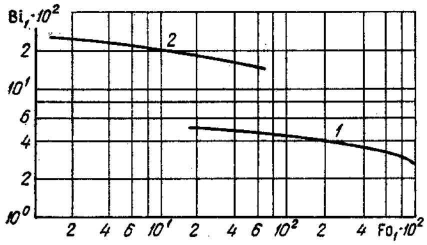

Fig. 3. Effect of the dimensions of interacting bodies on thermal stabilization: 1) $R_1 = R_2 = 0.1$ m; 2) $R_1 = R_2 = 0.5$ m.

arbitrary cross section $T_i(x, \tau)$ and of the resulting radiation density $E_1(0, \tau)$, has the form

$$T_i(x,\ \tau) = T_i + \frac{a_i}{\lambda_i R}\int_0^\tau \Big[1+2\sum_{n=1}^{\infty}(-1)^n \cos\frac{\pi n}{R}\times$$

$$\times\ [R+(-1)^{i+1}x]\exp[-\mu_i n^2(\tau-t)]\Big]\ E_i(0,\ t)\,dt, \quad (4)$$

$$E_1(0,\ \tau) = \sigma_{12}\Bigg\{\bigg(T_2 - \frac{a_2}{\lambda_2 R}\int_0^\tau \Big[1+$$

$$+2\sum_{n=1}^{\infty}\exp[-\mu_2 n^2(\tau-t)]\Big]E_1(0,\ t)\,dt\bigg)^4 -$$

$$-\bigg(T_1+\frac{a_1}{\lambda_1 R}\times$$

$$\times\int_0^\tau\Big[1+2\sum_{n=1}^{\infty}\exp\ [-\mu_1 n^2(\tau-t)]\Big]\ E_1(0,\ t)\ dt\bigg)^4\Bigg\}, \quad (5)$$

where $\mu_i = \pi^2 a_i/R^2$ $(i = 1, 2)$.

On converting (4) and (5) to dimensionless form we obtain

$$\theta_i(\xi,\ \tau) = 1+\eta_i\int_0 \Big\{1+2\sum_{n=1}^{\infty}(-1)^n\cos\pi n\ \times$$

$$\times\ (1+(-1)^{i+1}\xi)\times\exp[-\mu_i n^2(\tau-t)]\Big\}\ \varphi(t)\,dt, \quad (6)$$

$$\varphi(\tau) = \delta_2\Big\{1-\eta_2\int_0^\tau\Big[1+2\sum_{n=1}^{\infty}\exp\,[-$$

$$-\mu_2 n^2(\tau-t)]\Big]\times\varphi(t)\,dt\Big\}^4 -$$

$$-\delta_1\Big\{1+\eta_1\int_0^\tau\Big[1+2\sum_{n=1}^{\infty}\times$$

$$\times\exp[-\mu_1 n^2(\tau-t)]\Big]\ \varphi(t)\,dt\Big\}^4. \quad (7)$$

Here

$$\theta_i = \frac{T_i(\xi,\ \tau)}{T_i};\quad \varphi(\tau) = \frac{E_1(0,\ \tau)}{\sigma_{12}(T_2^4 - T_1^4)},$$

$$\delta_i = \frac{T_i^4}{T_2^4 - T_1^4};\quad \eta_i = \frac{a_i\sigma_{12}}{\lambda_i R}\,\frac{T_2^4 - T_1^4}{T_i},$$

$$\xi = \frac{x}{R}\ (i = 1,\ 2).$$

The nonlinear integral equation (7), which is shown in discrete form, reduces to a system of nonlinear algebraic equations in $\varphi(\tau)$ and is solved numerically with Wegstein's method [2], a modification of the familiar Newton method. The values for the dimensionless resulting radiation density $\varphi(\tau)$ obtained in this manner are then used in expression (6) to calculate the dimensionless temperatures $\theta_i(\xi, \tau)$ in any plate cross section. All calculations were performed on a computer.

Figure 2 shows the results obtained from calculating the dimensionless values of the resulting radiation density $\varphi(Fo_1)$ and the temperature $\theta_1(Fo_1)$ as functions of the Fourier number $Fo_1 = (\lambda_1/c_1\rho_1)(\tau/R^2)$; this applies to the case of nonstationary radiative interaction of plates ($R_1 = R_2 = R = 0.01$ m, $a_1 = \lambda_1/c_1\rho_1 = 0.324$ m^2/hr, $T_1 = 300°$ K, $a_2 = \lambda_2/c_2\rho_2 = 0.045$ m^2/hr, $T_2 = 1500°$ K), whose resulting emissivity is $\sigma_{12} = 5.67 \cdot 10^{-8}$ W/m^2 $\cdot$ deg^4.

Calculation results are also given for the interaction of the same plates but whose surfaces possess a much lower emissivity $\sigma_{12} = 3.97 \cdot 10^{-9}$ W/m^2 $\cdot$ deg^4. Compared to the second case (curves 2 and 4), thermal stabilization takes place within a much shorter time interval in the first case (curves 1 and 3).

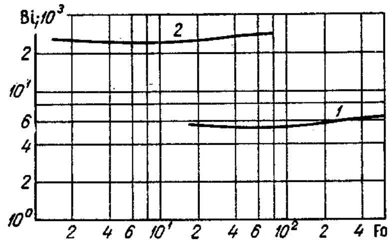

Fig. 4. Effect of the dimensions of interacting "thin" bodies on thermal stabilization: 1) $R_1 = R_2 = 0.01$ m; 2) $R_1 = R_2 = 0.05$ m.

We use the results obtained to find the value of an arbitrary coefficient of radiative heat transfer. Similarly to the coefficient of convective heat transfer (which also is a purely arbitrary concept) the coefficient of radiative heat transfer is defined by the relationship

$$\alpha(\tau) = \frac{E(0,\ T)}{T_2(\tau) - T_1(\tau)} = \frac{\sigma_{12}\left[T_2^4(\tau) - T_1^4(\tau)\right]}{T_2(\tau) - T_1(\tau)}. \quad (8)$$

We use $\alpha(\tau)$ to convert the results (similar to those obtained above) for $a_1 = 0.612$ m^2/hr, $T_1 = 300°$ K, $a_2 = 0.0636$ m^2/hr, $T_2 = 1500°$ K, and $\sigma_{12} = \sigma_0$; we then obtain the criterial dependence $Bi = f(Fo)$. Here $Bi = \alpha R/\lambda$ is the Biot number, constructed with respect to one of the interacting bodies. As is clear (Fig. 3), there is a characteristic relationship reflecting the decrease in values of Bi_1 and, consequently, of α_1 for a heated body with time (Fo_1). As the dimensions of the interacting bodies increase, the thermal stabilization processes are retarded somewhat. The nature of the relationship $Bi_1 = f(Fo_1)$ is unusual for relatively "thin" bodies (Fig. 4). In this case, an extremum is present on the curve $Bi_1 = f(Fo_1)$, whose position shifts toward larger Fo_1 as the dimension of the interacting plates increases. This feature is due to α and, consequently to, Bi_1. It is apparently explained by "reflection" of the thermal fluxes from an insulated surface of the plate. This effect becomes particularly noticeable for thin plates, as well as for plates with high thermal conductivity.

On the whole the weak dependence of the Biot number on time in almost the entire range of the thermal stabilization process is notable. Despite the fact that strong radiative interaction is clearly defined in the initial stage of thermal stabilization (see Fig. 2), the kinetics of change in the Biot number are in the nature of a sluggish and prolonged process. Here, the rate of decrease in Bi_1 is less in the initial stage than in the later stage.

The above discussion explains to a certain degree why the widely used quasi-stationary methods of calculating nonstationary heat transfer result in satisfactory agreement with experiment.

At the same time the preliminary results presented above provide a basis for performing more detailed determinations of $\alpha(\tau)$ when there is a distinct thermal interaction.

NOTATION

a_i denotes the thermal diffusivity coefficients; τ is the time; E_i denotes the resulting radiation densities, $i = 1, 2$; $\sigma_{12} = \sigma_{21}$ are the resulting emissivities of the plate; λ_i denotes the thermal-conductivity coefficients for the plate; R_i denotes the plate thickness; T_i is the initial plate temperature; ξ is a dimensionless coordinate; θ_i is the dimensionless temperature; $\varphi(\tau)$ is the dimensionless resulting radiation density; $Fo_i = (\lambda_i/c_i\rho_i)(\tau/R^2)$ is the Fourier number; $Bi_i = \alpha_i R_i/\lambda_i$ is the Biot number.

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THERMAL CONDITIONS FOR PRODUCING ARTICLES FROM A MELT

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The temperature distribution is indicated through directly measurable values from consideration of convection equations for the steady-state process. The impossibility of stabilization of the interface when the extraction rate changes is shown.

A. V. Stepanov has proposed a method for producing articles directly from a melt [1].

Various articles made from a number of materials are now being produced with this method [2, 3]. Which basically is as follows. The melt column is given the desired shape (see figure), and finished article is obtained by crystallization of the column. The importance of calculation of the position of the crystallization front is obvious here. This problem is also of interest for the Czochralski method.

The thermal conditions of the process for a specified transition-boundary position are calculated in this article in an approximation of a one-dimensional thermal problem with convection.

The cooling schemes can vary [3]. We assume that heat transfer occurs only due to internal thermal conductivity. The coordinate system is shown in the figure. We ignore change in the physical characteristics of the material on either side of the phase interface.

The equations of the problem are [4]

$$\frac{\partial \vartheta_1}{\partial t} + u\,\frac{\partial \vartheta_1}{\partial x} - a^2\frac{\partial^2 \vartheta_1}{\partial x^2} = 0, \quad 0 \leqslant x \leqslant X(t), \tag{1}$$

$$\frac{\partial \vartheta_2}{\partial t} + u\,\frac{\partial \vartheta_2}{\partial x} - a^2\frac{\partial^2 \vartheta_2}{\partial x^2} = 0, \quad X(t) \leqslant x. \tag{2}$$

At the boundary, we have

$$\vartheta_1(X,\ t) = \vartheta_2(X,\ t) = T_0, \tag{3}$$

$$K\left(\frac{\partial \vartheta_2}{\partial x} - \frac{\partial \vartheta_1}{\partial x}\right)\Big|_{x=X} = Lu\rho + L\rho\,\frac{dX}{dt}, \tag{4}$$

$$\vartheta_1(0,\ t) = \tau_m.$$

The simplest case of the stabilization problem consists, with steady-state extraction and therefore with a fixed interface, of maintaining the position of